Attachment #4

From these equations we find that

$$C_1 = -\frac{(p_0 - p_L)b}{2L}\left(\frac{\mu^{\mathrm{I}} - \mu^{\mathrm{II}}}{\mu^{\mathrm{I}} + \mu^{\mathrm{II}}}\right) \tag{2.5–15}$$

$$C_2^{\mathrm{I}} = +\frac{(p_0 - p_L)b^2}{2\mu^{\mathrm{I}}L}\left(\frac{2\mu^{\mathrm{I}}}{\mu^{\mathrm{I}} + \mu^{\mathrm{II}}}\right) = C_2^{\mathrm{II}} \tag{2.5–16}$$

Hence the momentum flux and velocity profiles are

$$\boxed{\tau_{xz} = \frac{(p_0 - p_L)b}{L}\left[\left(\frac{x}{b}\right) - \frac{1}{2}\left(\frac{\mu^{\mathrm{I}} - \mu^{\mathrm{II}}}{\mu^{\mathrm{I}} + \mu^{\mathrm{II}}}\right)\right]} \tag{2.5–17}$$

$$\boxed{v_z^{\mathrm{I}} = \frac{(p_0 - p_L)b^2}{2\mu^{\mathrm{I}}L}\left[\left(\frac{2\mu^{\mathrm{I}}}{\mu^{\mathrm{I}} + \mu^{\mathrm{II}}}\right) + \left(\frac{\mu^{\mathrm{I}} - \mu^{\mathrm{II}}}{\mu^{\mathrm{I}} + \mu^{\mathrm{II}}}\right)\left(\frac{x}{b}\right) - \left(\frac{x}{b}\right)^2\right]} \tag{2.5–18}$$

$$\boxed{v_z^{\mathrm{II}} = \frac{(p_0 - p_L)b^2}{2\mu^{\mathrm{II}}L}\left[\left(\frac{2\mu^{\mathrm{II}}}{\mu^{\mathrm{I}} + \mu^{\mathrm{II}}}\right) + \left(\frac{\mu^{\mathrm{I}} - \mu^{\mathrm{II}}}{\mu^{\mathrm{I}} + \mu^{\mathrm{II}}}\right)\left(\frac{x}{b}\right) - \left(\frac{x}{b}\right)^2\right]} \tag{2.5–19}$$

These distributions are shown in Fig. 2.5–1. Note that if $\mu^{\mathrm{I}} = \mu^{\mathrm{II}}$ both velocity distributions are the same and the results simplify to the parabolic velocity profile for laminar flow of a pure fluid in a slit.

The *average velocity* in each layer may now be calculated:

$$\langle v_z^{\mathrm{I}}\rangle = \frac{1}{b}\int_{-b}^{0} v_z^{\mathrm{I}}\,dx = \frac{(p_0 - p_L)b^2}{12\mu^{\mathrm{I}}L}\left(\frac{7\mu^{\mathrm{I}} + \mu^{\mathrm{II}}}{\mu^{\mathrm{I}} + \mu^{\mathrm{II}}}\right) \tag{2.5–20}$$

$$\langle v_z^{\mathrm{II}}\rangle = \frac{1}{b}\int_{0}^{b} v_z^{\mathrm{II}}\,dx = \frac{(p_0 - p_L)b^2}{12\mu^{\mathrm{II}}L}\left(\frac{\mu^{\mathrm{I}} + 7\mu^{\mathrm{II}}}{\mu^{\mathrm{I}} + \mu^{\mathrm{II}}}\right) \tag{2.5–21}$$

From the velocity and momentum-flux distributions given above, one can in addition calculate the maximum velocity, the velocity at the interface, the plane of zero shear stress, and the drag on the walls of the slit.

§2.6 CREEPING FLOW AROUND A SOLID SPHERE[1,2]

In the preceding sections several elementary viscous flow problems have been solved by making differential momentum balances. In the introductory section of this chapter it was emphasized that this method of analysis is

[1] See C. G. Stokes, *Trans. Cambridge Phil. Soc.*, 9, 8 (1850). Also H. Lamb, *Hydrodynamics*, Dover, New York (1945), First American Edition, §338, pp. 602 *et seq.*; V. L. Streeter, *Fluid Dynamics*, McGraw-Hill, New York (1948), pp. 235–240. A more detailed discussion is to be found in H. Villat, *Leçons sur les fluides visqueux*, Gauthier-Villars, Paris (1943), Chapter 7, in which the unsteady motion of a sphere is considered.

[2] Non-Newtonian flow around spheres has been studied by J. C. Slattery, doctoral thesis, University of Wisconsin (1959).

Attachment #4 cont'd

restricted to flow systems with straight streamlines. Because the problem of flow around a sphere involves curved streamlines, it cannot be solved by the techniques introduced in this chapter. Nevertheless, a brief discussion is appropriate here because of the importance of flow around submerged objects in engineering. No attempt is made to derive the expressions for the distributions of momentum flux, pressure, and velocity. We simply state these results and then proceed to use them to derive some important relations, which we shall need in later discussions.

Let us consider the very slow flow of an incompressible fluid about a solid sphere, as shown in Fig. 2.6–1. The sphere has a radius R and a diameter D. The fluid has a viscosity μ and density ρ and approaches the sphere vertically upward along the negative z-axis with uniform velocity v_∞. For very slow flow, the momentum flux distribution, pressure distribution, and velocity components in spherical coordinates have been found analytically to be

$$\boxed{\tau_{r\theta} = \frac{3}{2}\frac{\mu v_\infty}{R}\left(\frac{R}{r}\right)^4 \sin\theta} \tag{2.6–1}$$

$$\boxed{p = p_0 - \rho g z - \frac{3}{2}\frac{\mu v_\infty}{R}\left(\frac{R}{r}\right)^2 \cos\theta} \tag{2.6–2}$$

$$\boxed{v_r = v_\infty\left[1 - \frac{3}{2}\left(\frac{R}{r}\right) + \frac{1}{2}\left(\frac{R}{r}\right)^3\right]\cos\theta} \tag{2.6–3}$$

$$\boxed{v_\theta = -v_\infty\left[1 - \frac{3}{4}\left(\frac{R}{r}\right) - \frac{1}{4}\left(\frac{R}{r}\right)^3\right]\sin\theta} \tag{2.6–4}$$

In Eq. 2.6–2 the quantity p_0 is the pressure in the plane $z = 0$ far away from the sphere, $-\rho g z$ is the contribution of the fluid weight (hydrostatic effect), and the term containing v_∞ results from the flow of the fluid around the sphere. These equations are valid only for "creeping flow," which for this system occurs when the Reynolds number $Dv_\infty\rho/\mu$ is less than about 0.1. This region is characterized by the virtual absence of eddying downstream from the sphere.

Note that the velocity distribution satisfies the condition that $v_r = v_\theta = 0$ at the surface of the sphere. Furthermore, it may be shown that v_z approaches v_∞ for distances far from the sphere. In addition, the pressure distribution clearly reduces to the hydrostatic equation $p = p_0 - \rho g z$ far from the sphere surface. Hence the expressions do satisfy the boundary conditions at $r = R$ and $r = \infty$.

Let us now calculate the net force exerted by the fluid on the sphere. This force is computed by integrating the normal force and tangential force over the sphere surface.